[7.1] $P(3,1,4)$, $\alpha: x-2y+2z+3=0$

$$d = \frac{|3-2+8+3|}{\sqrt{1+4+4}}$$

$$= \frac{12}{3}$$

$$\therefore d = 4$$

[7.2] $O(0,0,0)$, $\alpha: 3x+2y-4z+5=0$

$$d = \frac{|0+0+0+5|}{\sqrt{9+4+16}}$$

$$\therefore d = \frac{5}{\sqrt{29}}$$

[8] $\alpha: 2x-y+z=3 \Rightarrow \vec{n}_\alpha = \langle 2,-1,1\rangle$

$\beta: x+2y-4z=4 \Rightarrow \vec{n}_\beta = \langle 1,2,-4\rangle$

Let $\vec{u} = \langle u_1,u_2,u_3\rangle$ be direction vector of ℓ. Since $\ell \perp \vec{n}_\alpha$ and $\ell \perp \vec{n}_\beta$,

$$\langle 2,-1,1\rangle \cdot \langle u_1,u_2,u_3\rangle = 2u_1 - u_2 + u_3 = 0$$

$$\langle 1,2,-4\rangle \cdot \langle u_1,u_2,u_3\rangle = u_1 + 2u_2 - 4u_3 = 0$$

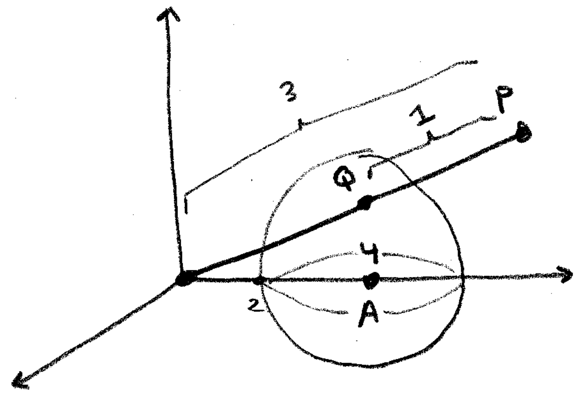
$$\begin{bmatrix} 2 & -1 & 1 & 0 \\ 1 & 2 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2/5 & 0 \\ 0 & 1 & -9/5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{matrix} u_3 = t \\ u_2 = \frac{9}{5}t \\ u_1 = \frac{2}{5}t, \end{matrix}$$

so $\vec{u} = 5t\langle 2,9,5\rangle$ and the direction of ℓ is $\langle 2,9,5\rangle$ $\square$

which we knew from symmetric EQNS of ℓ in demo 3.

P 163

[1]



P divides OQ externally in ratio 3 : 1

Let $\vec{p}, \vec{q}, \vec{a}$ be position vectors of P, Q, A.

(1) $\quad |\vec{q} - \vec{a}| = 2$

$$\vec{p} = \frac{3\vec{q} - 1\vec{o}}{3-1}$$

(2) $\quad \vec{p} = \frac{3}{2}\vec{q} \equiv \vec{q} = \frac{2}{3}\vec{p}$

(1,2) ⇒ (3) $\quad |\frac{2}{3}\vec{p} - \vec{a}| = 2$

$$\frac{2}{3}|\vec{p} - \frac{3}{2}\vec{a}| = 2$$

$$|\vec{p} - \frac{3}{2}\vec{a}| = 3,$$

But this is equation of circle center $\vec{c} = \frac{3}{2}\langle 0, 4, 0\rangle = \langle 0, 6, 0\rangle$ and radius 3

∴ Figure traced by P is Circle center $\langle 0, 6, 0\rangle$ radius 3.

p 163, ctd

[2] $\vec{a} = \langle 4, -6, 8 \rangle$. sphere $|2\vec{p} - 3\vec{a}| = 6$.

$$|2\vec{p} - 3\vec{a}| = 6$$

$$2|\vec{p} - \tfrac{3}{2}\vec{a}| = 6$$

$$|\vec{p} - \tfrac{3}{2}\vec{a}| = 3$$

$$\tfrac{3}{2}\langle 4, -6, 8 \rangle = \langle 6, -9, 12 \rangle$$

$\therefore$ Center at $\langle 6, -9, 12 \rangle$, radius $= 3$